November 22, 1883.

Professor T. H. HUXLEY, President, in the Chair.

In pursuance of the Statutes, notice was given from the Chair of the ensuing Anniversary Meeting, and the list of Officers and Council nominated for election, was read as follows:—

President.—Professor Thomas Henry Huxley, LL.D.

Treasurer.—John Evans, D.C.L., LL.D.

Secretaries.— Professor George Gabriel Stokes, M.A., D.C.L., LL.D.
Professor Michael Foster, M.A., M.D.

Foreign Secretary.—Professor Alexander William Williamson, LL.D.

Other Members of the Council.—Captain W. de Wiveleslie Abney, R.E.; Professor W. Grylls Adams, M.A., F.C.P.S.; the Duke of Argyll, K.T., D.C.L.; John Gilbert Baker, F.L.S.; Thomas Lauder Brunton, M.D., Sc.D.; William Henry M. Christie, Astron. Royal; Warren De La Rue, M.A., D.C.L.; Sir Frederick J. O. Evans, K.C.B.; Professor George Carey Foster, B.A.; Francis Galton, M.A., F.G.S.; James Whitbread Lee Glaisher, M.A.; Sir William Withey Gull, Bart., M.D.; Hugo Müller, Ph.D.; Professor Joseph Prestwich, M.A., F.G.S.; Professor Osborne Reynolds, M.A.; Osbert Salvin, M.A., F.L.S.

The Presents received were laid on the table, and thanks ordered for them.

The following Papers were read:—

I. "On the Formation of Ripple-mark in Sand." By G. H. DARWIN, F.R.S., Plumian Professor and Fellow of Trinity College, Cambridge. Received October 18, 1883.

The following paper contains an account of experiments and observations on the formation of ripple-mark in sand. The first section is devoted to experiments on the general conditions under which ripple-mark is formed, and especially on the mode of formation and maintenance of irregular ripples by currents. In the second section it is shown that regular ripple-mark in sand is due to a complex arrangement of vortices in oscillating water; and the last section gives some account of the views of certain recent observers in this field, and a

discussion of some phenomena in the vortex motion of air and water.

§ 1. *First Series of Experiments.*

A cylindrical zinc vessel, like a flat bath, with upright sides, 2 feet 8 inches in diameter and 9 inches deep, was placed on a table, which was free to turn about a vertical axis. Some fine sand was strewn over the bottom to a depth of about an inch, and water was poured in until it stood three inches deep over the sand. After some trials of simply whirling the bath, in which no regular ripple-mark was formed, I found that rotational oscillation with a jerking motion of small amplitude gave rise almost immediately to beautiful radial ripples all round the bath. If the jerks were of small amplitude the ripples were small, and if larger they were larger. On one occasion having made large ripple-marks, I oscillated the bath much more rapidly, and a second set of ripples sprang into existence in the furrows of the first set. Another time, when in consequence of irregularity in the motion, a set of radiating waves were generated in the water, a second set of transverse ripples were formed, which produced by interference a beautifully mamellated structure, arranged like a chess-board. In all these experiments the radiating ripples began first to appear at the outer margin of the bath and grew inwards; but the growth stopped after they had extended to a certain distance. If the jerking motion was violent, ripples were not formed near the circumference, and they only began at some distance inwards. After these preliminary trials, arrangements were made for regularising both the frequency and amplitude of oscillation of the bath. An attempt was then made to formulate the laws which govern the generation of ripple-marks. In the following notes of experiments the expression " octave " is used to denote a ripple-length which is one-half of the main or fundamental ripple, and the amplitudes are measured by the displacement of the edge of the bath.

The water stood 1 inch deep in the bath.

1. Amplitude 1 inch; frequency 52 per minute (complete oscillations).

No ripples formed after four minutes.

2. Amplitude $2\frac{1}{2}$ inches; frequency 52.

55 ripples round the circumference, extending about 4 inches inwards; somewhat irregular and with a tendency to break into the octave.

3. Amplitude $6\frac{1}{2}$ inches; frequency 52.

Motion very violent. About 5 large irregular ripples in the circumference, breaking at about 9 inches from the outside into about 40 ripples.

4. Amplitude $1\frac{3}{4}$ inches; frequency 52.

In one part 7 ripples to 9 inches; at another part 12 ripples to 1 foot; some tendency to break into the octave. The ripples only extended an inch or two from the edge.

More water was then poured in until it stood about $1\frac{3}{4}$ inches deep.

5. Repetition of No. 4.

There were 33 ripples in a half circumference (which is 22 inches); the ripples were more regular than in No. 4, with not so much tendency to break into the octave.

6. Water $2\frac{1}{2}$ inches deep; frequency 59; amplitude 3 or 4 inches.

43 ripples to the circumference (which is 44 inches), extending inwards 8 inches.

Hereafter the amplitudes were marked by a pointer projecting 3 inches from the edge of the bath.

7. Water $2\frac{1}{2}$ inches; frequency 75; amplitude $2\frac{1}{2}$ inches.

66 ripples to circumference, very regular, and extending inwards 4 or 5 inches.

8. Water $2\frac{1}{2}$ inches; frequency 74; amplitude 3 inches.

63 or 64 ripples, extending 6 or 7 inches; broken in two or three places.

9. Water $2\frac{1}{2}$; frequency 75; amplitude 4 inches.

53 ripples, extending 8 or 9 inches; not so regular.

10. Water $2\frac{1}{2}$; frequency 78; amplitude 5 inches.

47 ripples, extending 10 or 11 inches.

11. Water $2\frac{1}{2}$; frequency 75; amplitude 6 inches.

Agitation violent; all the coarser sand collected round the margin, without ripple-mark for 4 inches inwards; from 4 to 11 inches inwards, rather irregular ripples about 37 to circumference; the usual flat centre.

12. Water $2\frac{1}{2}$; frequency about 80; amplitude 7 inches.

The water churned up the sand with violence; margin the same as No. 11; from 8 to 12 inches from margin rather irregular ripples, about 34 to circumference.

13. (Bad observation.) Water $2\frac{1}{2}$; frequency about 85; amplitude about $1\frac{1}{2}$ to 2 inches.

80 ripples to circumference.

14. (Bad observation.) Water $2\frac{1}{2}$; frequency 57; amplitude 2 inches.

No ripples raised.

An analysis of the observations marked 7 to 14 was made on the hypothesis that the water remained still, when the bath oscillated with a simple harmonic motion. I endeavoured to find whether λ, the wave-length of ripple (in inches) was directly proportional to v, the maximum velocity of the water relatively to the bottom (in inches per minute) during the oscillatory motion; also the values of v_1 and

v_2, the least and greatest velocities of the water compatible with the formation of ripple-mark.

The following are the results :—

$\lambda \div v$.		Feet per sec. v_1.		Feet per sec. v_2.
·0031	....	·51 to ·56	....	—
·0027 to ·0028	....	·46 to ·51	....	—
·0024	....	·43 to ·49	....	—
·0021	....	·43 to ·52	....	—
·0023	....	·50	....	1·2
·002	....	·56	....	1·12
—	....	more than ·42	....	—
Mean ·00245 min. or ·147 second.	....	·503	....	1·2

It appears therefore that ripples are not formed if the maximum velocity of the water relatively to this particular sand, estimated on the above hypothesis, is less than half a foot or greater than a foot per second; and that if v be that maximum velocity in inches per minute, the wave-length of ripples generated is ·00245v, or ·147v when v is measured in inches per second. The results seem as fairly consistent with one another as could be expected. It will appear from section 2 that the maximum velocity of the water, as estimated on the hypothesis that the water as a whole executes a simple harmonic oscillation relatively to the bottom, does not give the maximum velocity of the water in contact with the sand relatively thereto. The quantity called v is not in reality the maximum velocity of the water in contact with the bottom relatively thereto, but it is in fact 6·283 times the amplitude multiplied by the frequency. Thus we cannot conclude that a current of half a foot per second is just sufficient to stir the sand. In the state of oscillation corresponding to v_1, it is probable that part of the water at the bottom is moving with a velocity much greater than half a foot per second relatively to the sand. The number of the experiments analysed is insufficient for the accurate determination of the law connecting wave-length, amplitude, and frequency; but this branch of the subject was not pursued further because other observers, whose work is referred to in § 3, have made a number of experiments with this object.

It was after making this set of experiments that I hit on what appears to be the key-note of the whole phenomenon.

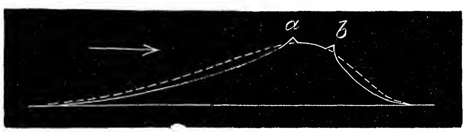
A series of ripples extending inwards for some distance having been made by oscillation, and the water having come to rest, the bath was turned slowly and nearly uniformly round. The ripples were then observed to prolong themselves towards the centre; this shows that a uniform current is competent to prolong existing ripples. The

uniform current flattened the tops of the ripples, but made the lee-side steeper. After being exposed to a prolonged current, the ripples were not only not in course of obliteration, but became somewhat more pronounced.

The sand was then smoothed with the edge of a board; after the exposure of the sand to a current, marks made with the edge, which at first were too faint to be seen, became by a course of development well-defined ripples. The whole surface became gradually mottled with irregular chains of ripples of which the weather-side was a very gradual slope, and the lee-side was steep. The appearance was strikingly like that of drifted snow. As it might be conjectured that there would be eddies or vortices on the lee-side, I made some regular ripple-marks by oscillation, and then exposed them to a current. I shortly observed minute particles lying on the surface of the sand climbing up the lee-slope of the ripples apparently *against* stream. This proved conclusively the existence of the suspected vortices.

If when the bath was at rest, a sudden motion was given in one direction, the sand on the lee-side of each ripple was observed to be churned up by a vortex. By giving a short and sudden motion, I was able to see the direct stream pile up the sand on the weather-side and the vortex on the lee-side. Fig. 1 shows the effect of a

Fig. 1.



single short jerk. Two little parallel ridges of sand were formed, namely (*a*) by the direct stream, and (*b*) on the lee-side by the vortex, a little below the crest of the ripple-mark.

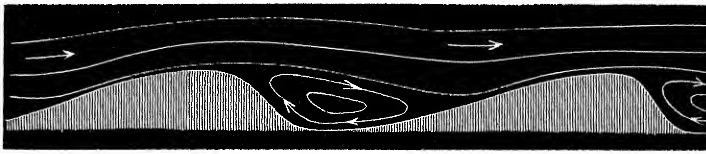
It is thus clear that casual surface inequalities are accentuated by the combined action of the direct stream and of the vortex.

For the purpose of examining the vortices a glass tube was drawn out to a fine point, and fitted at the other end with a short piece of india-rubber tube. With this a drop of ink could be squirted out at the bottom of the water. This method was adopted in all subsequent observations, and it proved very valuable. It may be worth mentioning that common ink, which is heavier than water, was better than aniline dye; and the addition of some sulphate of iron to the ink improved its action.

A drop of ink was placed in the furrow between two ripples; as soon as the continuous stream passed, the ink was parted into two portions, one being sucked back apparently against stream up the lee-side of the ripple-mark, and the other being carried by the direct

stream towards the crest. By observing the limits of the transport of the ink, I conclude that the stream lines were as shown in fig. 2.

FIG. 2.



These points being settled, it remained to discover how the vortices were arranged which undoubtedly must exist in the oscillatory formation of regular ripples. The rapidity of the necessary oscillations made this a task of some difficulty.

§2. *On the Formation of Ripple-mark by Oscillation.*

The observations were made in two different ways.

In the first of these, which also ultimately proved to be the most successful, the ripple-mark was made in a glass trough about 1 foot long, 5 inches wide, and 6 inches deep. In order to observe the formation of the ripples absolutely in profile, a sheet of glass fitting the trough was placed to stand on four short corks at the bottom. The trough was then put to rock on two corks, one at each side, on the line which bisects its length. Two other slightly shorter corks were put at the ends; these served as stops, and only allowed it to rock through a very small angle. The trough was placed on a window-sill with a strong light outside, and was gently rocked by hand.

When the trough is half filled with water, and sand is sprinkled on the glass plate, it is easy to obtain admirable ripple-marks by gently rocking the trough.

When a very small quantity of sand is sprinkled in and the rocking begins, the sand dances backwards and forwards on the bottom, the grains rolling as they go.

Very shortly the sand begins to aggregate into irregular little flocculent masses, the appearance being something like that of curdling milk. The position of the masses is, I believe, solely determined by the friction of the sand on the bottom, and as soon as a grain sticks, it thereby increases the friction at that place.

The aggregations gradually become elongated and rearrange themselves. As soon as the formation is definite enough to make the measurement of the wave-length possible, it is found that the wave-length is about half of what it becomes in the ultimate formation.

Some of the elongated patches disappear, and others fuse together and form ridges, the ridges then become straighter, and finally a regular ripple-mark is formed with the wave-length double that in the initial stage.

When a drop of ink is put on the glass without any sand, it simply slides to and fro, with perhaps a faint tendency to curdle, but it cannot be caused to form ripple-mark. This shows that the initial stage when the sand is beginning to curdle is due simply to friction.

When the ink is put upon a flocculent mass it betrays some kind of dance in the water, but the layer of water disturbed is so thin that it would hardly have been possible to detect the law of the motion from this case alone. When, however, the nature of that motion, as described below, has been discovered, the same kind of motion may be recognised in the dance of the ink over these flocculent aggregations.

I found in the later experiments that it was advantageous to have a very regular ripple-mark. I therefore sprinkled sand on the sheet of glass, and, before beginning the rocking, I traced regular furrows in it with the point of my finger. A few oscillations of the trough soon effaced all signs of the artificial origin, and the ripple crests with the bare glass in the furrows, were absolutely indistinguishable, except by perfect regularity, from those produced naturally. Most of the observations were, however, made with the natural ripples, and it was only towards the end that I adopted this plan in order to save time and to obtain perfect regularity.

In the rocking trough, the water moves whilst the bottom of the vessel is still, save for the small rocking motion. A second arrangement was, however, made in which the converse is true. A sheet of plate glass is caused to oscillate in the bottom of a trough with glass sides. The oscillator is moved by a connecting rod and crank driven by a small water-motor, the throw of the crank is small, and the rapidity of oscillation can be varied within considerable limits.

When sand is sprinkled on the oscillating sheet of glass, phenomena such as described above are again observed, and good regular ripple-mark is formed in the sand. Although much was learned from this instrument, still the rocking trough was on the whole more useful.

It appeared to be certain from the first set of experiments that ripple-mark was due to eddies or vortices, and the question remained as to how the vortices were arranged in oscillatory motion. It required some practice, and many hours of watching, to establish the conclusions explained below, indeed the phenomena next described were only detected long after that which follows them in this paper.

If a very gentle oscillation be started, the layer of ink on the crest of a ripple-mark becomes thicker and thinner alternately, swaying backwards and forwards; then a little tail of ink rises from the

crest, and the point of growth oscillates on each side of the crest; the end of the tail flips backwards and forwards. Next the end of the tail spreads out laterally on each side, so that a sort of mushroom of ink is formed, with the stalk dancing to and fro. The height of the mushroom is generally less than a millimetre.

Fig. 3.

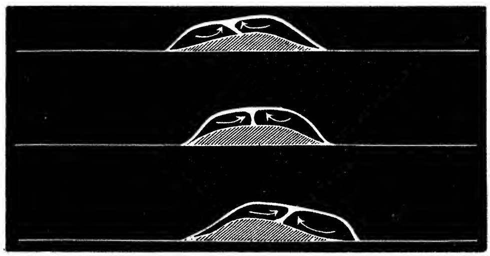


Fig. 3 is the best representation I can make of this appearance, which I shall call an ink mushroom. The first of these figures gives the extreme of excursion on one side, the second the mean position, and the third the extreme on the other side. The figures show the state of affairs when the oscillation is very gentle, so that the amplitude of oscillation of the main body of water is small compared with the wave-length of the ripple-mark. The elongated hollows under the mushroom are the vortices, and the stem is the upward current. If the ink be thick these spaces are clouded, and the appearance is simply that of an alternate thickening and thinning of the ink on the crest. When one is familiar with this motion, after examining it carefully with gentle oscillation over ripple-mark of some size, the same kind of dance may, I think, be detected in the stage of ripple manufacture after the sand has curdled into elongated flocculent masses.

The oscillations being still gentle, but not so gentle as at first, streams of ink from the two mushrooms on adjacent crests creep down the two slopes into the furrow between the adjacent ridges, and where they meet a column of ink begins to rise from the part of the water whose mean position is in the centre of the furrow. The column is wavy, and the appearance is strikingly like that of smoke rising from a fire in still air.

The column ascends to a height of some 5, 10, or perhaps 20 times the height of the ripple-marks, according to the violence of the agitation. It broadens out at the top on each side and spreads out into a cloud, until the appearance is exactly like pictures of a volcano in violent eruption; but the broad flat cloud dances to and

fro relatively to the ascending column. The ink continues to spread out laterally and begins to fall on each side. In this stage if the ink is not thick it is often very like a palm-tree, and for the sake of a name I call this appearance an ink tree. The branches (as it were) then fall on each side, and the appearance becomes like that of a beech-tree, or sometimes of an umbrella. The branches reach the ground, and then creep inwards towards the stem, and the ink, which formed the branches, is sometimes seen ascending again in a wavy stream parallel to the stem.

Perhaps a dozen or twenty oscillations are requisite for making the ink go through the changes from the first growth of the tree.

The descending column of a pair of trees comes down on to the top of the mushroom. I have occasionally, when the oscillations are allowed to die, seen both tree and mushroom, but the successful manufacture of the tree necessitates an oscillation of sufficient violence to render the observation of the mushroom very difficult.

The alternate thickening and thinning of the ink on the crests seems to render it probable that with moderate oscillation the mushroom vortices are still in existence, or at any rate that alternately one and the other is there. With violent oscillation, when the stem of the tree is much convoluted, as described below, it cannot be asserted that the mushroom vortices exist, and I am somewhat inclined to believe them to be then evanescent.

Each side of the ink tree is clearly a vortex, and the stem is the dividing line between a pair, along which each vortex contributes its share to the ascending column of fluid. The vortex in half the tree is clearly in the first place generated by friction of the vortex in its correlated mushroom, and is of course endued with the opposite rotation. The ascending stem of the tree is a swift current, but over the mushroom the descending current is slow until close to the mushroom, when it is seen to be impelled by pulses.

I was on one occasion fortunate enough to observe a mote in the water which was floating nearly in the centre of a tree vortex, and counted twelve revolutions which it made before it was caught away from its fortunate position.

If the adjoining crests are of unequal height the stem of the tree is thrown over sideways away from the higher crest; and indeed it requires care to make the growth quite straight. The ink in the stem ascends with a series of pulses, and it is clear that there is a pumping action going on which renders the motion of each vortex somewhat intermittent, the two halves of the tree being pumped alternately.

The amount of curvature in the stem of the tree depends on the amplitude of the oscillation of the water. Figs. 4, 5, 6, give fair representations of ink trees.

FIG 4.

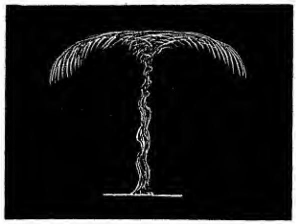


FIG 5.

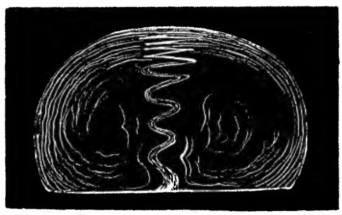


FIG. 6.

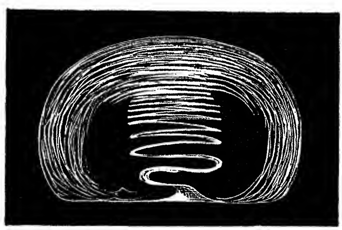


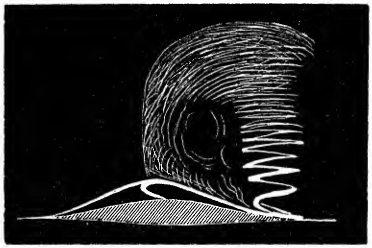
Fig. 4 is the palm-tree stage with gentle oscillation, and figs. 5 and 6 represent the appearance when the amplitude is greater.

Fig. 7 exhibits a tree in which the growth is one-sided on account of inequality between the heights of the bounding crests.

FIG. 7.

Fig. 8 represents a mushroom and a tree which I have occasionally succeeded in observing simultaneously.

Fig. 8.



The ink is propagated along the convolutions of the stem of the ink tree, but the convolutions are themselves propagated upwards, and each convolution corresponds to one oscillation. The motion of the ink along the convolutions soon becomes slow, but the convolutions become broader and closer. Thus the upper part of the tree is often seen to be most delicately shaded by a series of nearly equidistant black lines. A perfectly normal ink tree, made by a very thin stream of ink, would be like the fig. 9, in which the whole is formed by a

Fig. 9.

single line; but it is not possible to represent the extreme closeness of the lines adequately.

In the transition from the mushroom stage to the tree stage it appeared to me that it was very frequent that only half the ink tree was formed. At any rate I have frequently noted the mushrooms and half the tree vortices lasting during many oscillations, and then the other halves of the trees gradually appeared. This might, of course, be due to an accidental deficiency of ink in an invisible tree vortex, but I have observed this appearance frequently when there is ink at the stem of the tree, and when there seemed no reason why it should only be carried up in one ascending stream and not in the other.

If the agitation is very gentle the sand on the crests of the ripple-marks is just moved to and fro; with slightly more amplitude, the dance is larger, and particles or visible objects, such as minute air-bubbles, in the furrows, also dance, but with less amplitude than those on the crests. When the rocking is gentle the oscillation in the

furrow appears to be in a different phase from that on the crest, with more violent rocking I did not observe the difference of phase. The dance is not a simple harmonic motion like that of the main body of the water relatively to the bottom, but the particles dash from one elongation to the other, pause there, and then dash back again.

As the amplitude further increases the furrows are completely scoured out, and the sand on the crests is dashed to and fro, forming a spray dancing between two limits. With violent agitation this dance must have an amplitude of more than half a wave-length. If the agitation be allowed to subside the dance subsides, and when the water is still the ripple-mark is left symmetrical on both sides. With extremely violent oscillation all the water becomes filled with flying dust, and it is no longer possible to see what is happening. This seems to be the condition when the agitation is too strong for the formation of ripple-mark. It is probable that the rush of water sweeps away the existing ripple-mark, and there is then no longer anything to produce a systematic arrangement of vortices.

In fig. 10 I have tried to exhibit the dance of the vortices by the succession of figs. i to vii. When the amplitude of oscillation is the same as when the ripple-mark is generated, the series of changes is of the kind shown. The figures succeed one another in time, but they do not pretend to such accuracy as to represent the stages at rigorously equal intervals.

The dotted waves show a mean contour of the ripple-mark; they are introduced to show the displacements of the crests relatively to the mean position. A perfectly symmetrical ripple-mark does not, however, present a simple harmonic outline, for the hollows are flat and the crests rather sharp.

The convoluted line is the stem of the ink tree, which forms the dividing line of the two vortices; the curved arrows show the direction of rotation.

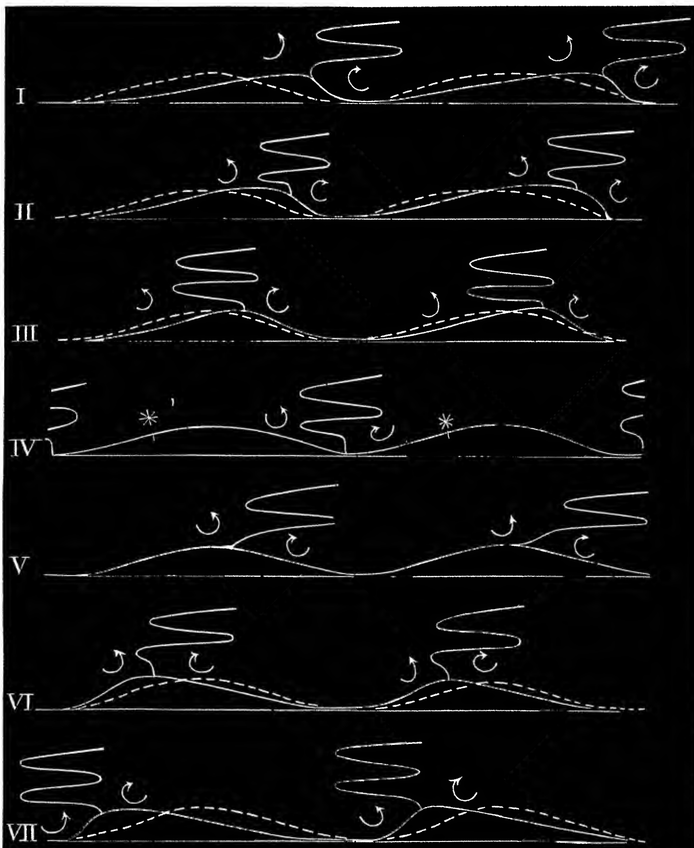
In i the water is at its elongation to the right. The crest of the ripple-mark is also at its extreme to the right, and the right hand slopes are steep, whilst the left are gentle. Here the water is at rest except for the vortices, which both tend to carry sand up to the crest.

In ii the general mass of water is beginning its movement to the left; it carries with it the upper convolutions of the ink tree, but leaves the root very nearly in the same position as in i. The crest of the ripple-mark is but little displaced. In iii the crest has begun its displacement, so that although the root of the ink tree has begun moving to the left, it is still over the crest. The convoluted stem continues to move to the left with increasing velocity, leaving the root behind it over the crest.

Just before the convolutions reach the position over the middle of

the furrow, the root leaves its crest and moves with very great speed to the left. In iv the root is just passing under the convolutions. The whole system is moving with its maximum velocity, but the root outstrips the stem. The two slopes of the crest are nearly symmetrical. In v the root has gained so far on the convolutions as to have again reached a crest.

Fig. 10.



In vi the convolutions have caught up the root, and the crests are being displaced.

Finally vii is a repetition of i in the opposite direction, and the half oscillation is completed.

If in these figures i to vii we take the wave-length of ripple-mark as unity, the amplitude of oscillation of the main body of water is 2·1,

that of the crests is ·7, and the breadth of the convolutions of the tree is ·3. The sum of the amplitude of oscillation of the crest, with the breadth of the convolutions, and the wave-length of ripple-mark is equal to 2, and this is very nearly equal to the amplitude of oscillation of the water, as it ought to be.

The law which governs the intensity of the vortices must be a matter of inference, since I found the motion too rapid to be sure of anything save that the vortices are driven alternately by pulses, and that the motion was most energetic near the elongations.

In i the right hand vortex of a pair must be at its maximum of intensity, and it seems probable that the left hand vortex has a sub-maximum in consequence of the friction of the water along the dividing line. During the return motion from i to vii, the left hand vortex must be increasing in intensity, so that it is at its maximum in vii. Probably the right hand vortex diminishes in intensity from i to v, and then increases to its sub-maximum in vii.

I am not able to say from observation that the vortices which have been described as giving rise to the ink mushrooms actually exist in this state of oscillation, but if they are there, one of them should be found at the point marked with an asterisk in iv.

The figures tell better than words the mechanism by which the ripple-mark is made and maintained, and the cause of the dance of the crests. The only difficulty is in stage iv, where the root of the tree is in the state of transferenee from one crest to the next. In this stage the vortices would seem to be in the act of degrading the ripple-mark, but they are not then either of them at their maximum of intensity, and the time during which this holds good is exceedingly short compared with the whole semi-period of oscillation. It seems somewhat likely that small vortices are called into existence at the points marked with asterisks in iv, which serve to protect the ripple-marks from degradation during the transference.

Fig. 11, i to vii, exhibits the dance of the vortices when the oscillation of the water is considerably less in amplitude than the wave-length of ripple-mark.

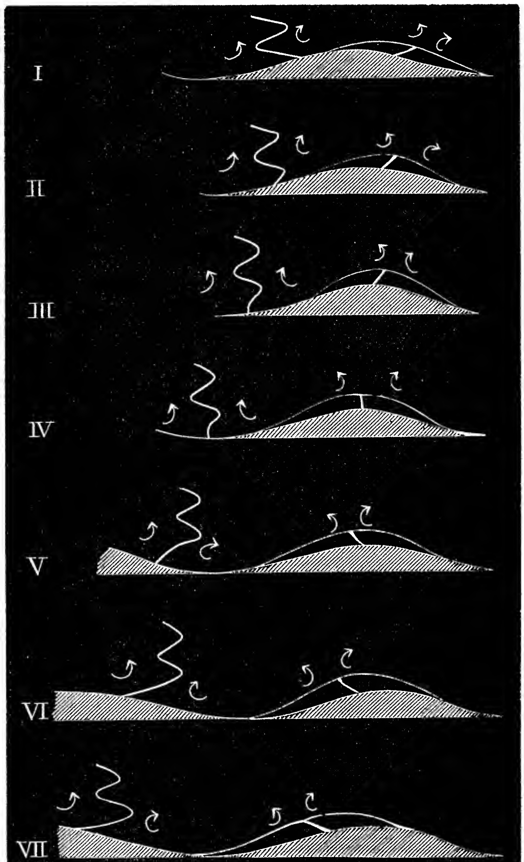
Here the crests of the ripple-mark are scarcely sensibly disturbed. Above the crest is drawn the pair of mushroom vortices, the curved arrows showing the direction of rotation being placed outside of the mushrooms; but I am not able to satisfy myself that they are both in existence during the whole oscillation. Fig. 8 above exhibits an appearance which I have sometimes seen, which seems to show that they may both exist together with an ink tree.

We must now draw attention to the manner in which the convolutions are added to the ink tree, and thus show the continuity of this fig. 11 for gentle oscillation with fig. 10 for violent oscillation.

In fig. 10, in ii, iii, iv, a convolution is added, which is unwrapped

again in v. It is again formed in vi and vii, and then becomes permanent, and is transmitted up the stem of the tree.

Fig. 11.



In fig. 11 the convolution is added in ii, and then remains permanently part of the tree; but a partial convolution is added in iii and iv, which is unwrapped again in v, vi, vii.

Thus in violent oscillation the convolution is permanently added just before an elongation, and in gentle oscillation afterwards. It would be easy to construct a figure for an intermediate amplitude in which the convolution is added just at elongation. If the oscillation gradually increases, the convolutions are permanently added sooner and sooner, and at the same time the formation of convolutions and subsequent unwrapping assumes more and more prominence.

It must be understood that these figures are drawn from the results of long watching of the process. My attention was at one time directed to one part of the phenomenon, and at another to a different part, and the amplitudes were constantly varied. I do not pretend to be able to see all these changes in a single half oscillation, lasting barely half a second. It may appear that I am incorrect in some parts of the construction, but I would ask any one who repeats the experiments not to condemn me hastily, for the constructions which I have given are the results of frequent trials and errors in the attempt to represent the changes observed.

I have not been able to determine exactly the mode of motion in the initial stages of ripple-making, when the oscillation has large amplitude, but when the ripple-marks are still in what I have described as the curdling stage.

If a current be passed over existing ripple-mark a vortex is established on the lee of each ripple; if the current be reversed the vortex is on the other side. Thus intermittent opposite currents will form ripple-mark, but probably without giving it a very regular wave-length.

If the intermittence is rapid, the vortex established on the lee-side, when the current is in one direction, is not annulled when the current is reversed, but it will be carried over the crest of the ripple-mark, and will diminish in intensity, whilst the new vortex with opposite rotation is established.

The study of a very gentle oscillation over existing ripple-mark, by means of a drop of ink placed on the ridge, enables us to observe these vortices (see fig. 3). I think it depends on the amplitude of oscillation whether both vortices are always in existence and simply vary in intensity, or whether the vortex due to motion to the right is quite annulled during the motion to the left, and *vice versâ*.

It may be suspected, therefore, that, in the early stages of ripple-making, when the amplitude of oscillation is large, vortices are set up in the lee of each aggregation of sand, in the same way as if the current were permanent, and that when the current is reversed these vortices are speedily annulled, and a new set on the other side of the aggregations is established. When a drop of ink is put on an aggregation, and the oscillation is started, the ink forms a layer of not more than half a millimetre in thickness. It is easy to see that there takes place some kind of rapid oscillation which is not simply harmonic. It appears to present all the characters of the motion when gentle oscillation is established over ripple-mark of some height, and therefore it is probable that the motion is of the same kind in both cases.

When the aggregations are more pronounced, small correlated tree vortices are set up. As above stated, it has seemed to me that frequently only half of each tree vortex is set up at first.

I am disposed to regard this as the transitional state from the mode of oscillation, which produces the octave with small height of ripple-crest, to the fundamental with considerable height.

In gentle oscillation over high ripple-mark the tree vortices are, in the first instance, seen to be started by the mushroom vortices, and the same is probably true of the condition we are considering.

If the suggested view as to the mode of transition be correct, then we must suppose that at first every alternate tree vortex is started by its correlated mushroom vortex. If there be no tree vortices, or if there be only every alternate one, the vortices can pack twice as close as if the trees are symmetrical; but the existence of a half tree vortex tends to generate its other half, and this half cannot exist normally unless every alternate ripple-mark is removed. The degradation of the alternate ripple-mark must arise then from the existence of the second half of the ink tree. In these early stages the phenomenon is not highly regular, and therefore, besides the smallness of the scale and the rapidity of the motion, we have the difficulty of irregularity to contend with.

Other observers have endeavoured to determine the laws connecting the wave-length in the ultimate formation with the various concomitant circumstances, and I shall leave this subject to the following section, where some account of their work will be given.

We may summarise the results of these observations as follows :—

The formation of irregular ripple-marks or dunes by a current is due to the vortex which exists on the lee of any superficial inequality of the bottom; the direct current carries the sand up the weather slope and the vortex up the lee slope. Thus any existing inequalities are increased, and the surface of sand becomes mottled over with irregular dunes. The velocity of the water must be greater than one limit and less than another, the limiting velocities being dependent on the average size and density of the particles. Existing regular ripple-mark is maintained by a current passing over it perpendicular to the ridges. A slight change in form ensues, the weather slope becoming less steep and the lee slope steeper. The ridges are also slowly displaced to leeward. The regular ripple-mark may also thus be somewhat prolonged, so that although a uniform current cannot, as I believe, form regular ripple-mark, yet it may increase the area over which it is to be found.

Regular ripple-mark is formed by water which oscillates relatively to the bottom. A pair of vortices, or in some cases four vortices, are established in the water; each set of vortices corresponds to a single ripple-crest and the vortices oscillate about a mean position, changing their shapes and intensities periodically, but not with a simple harmonic motion.

The successive changes in the vortex motion, whilst ripple-mark

is being established, and when the amplitude of oscillation over existing ripple-mark varies, are complex. As far as I have been able to determine, the following is an account of the phenomena:—

We begin with variation in amplitude of oscillation over existing regular ripple-mark, where the height of the undulations is not a very small fraction of the wave-length.

When the amplitude of oscillation is small compared with the wave-length, a pair of small vortices are established above the crest of each ripple-mark, rotating in opposite directions. In the mean position the upward current is over the crest, and the current of water tends to carry up sand from each furrow to the crest. The dividing line of the vortices oscillates, but the bottom of the line has much less amplitude of oscillation than the top, so that the dividing line is alternately inclined to one and the other side of the vertical. The vortices are thus carried backwards and forwards over the crest of the ripple, but the current always tends to maintain the crest, merely displacing very slightly the position of the highest point. The vortex which is on the lee-side is more intense than the other. We will call these the primary vortices. (See fig. 3.)

Suppose now the amplitude of oscillation to be somewhat larger; then the primary vortices by their friction on the adjacent water generate two other vortices. The upward current in these secondary vortices has its mean position over the middle of a furrow, and the current comes down immediately over the upward current of the primary pair of vortices. It appears that sometimes only every alternate one of the secondary vortices is established. The upward current of the secondary vortices oscillates with a motion which is very far from being harmonic. It remains at its elongation for a long time and then darts across to the other elongation. (See fig. 11.) During this mode of oscillation the primary vortices are carried much further backwards and forwards over the crests.

With still larger amplitude of oscillation it is no longer possible to distinguish the primary vortices, and the secondary vortices increase in intensity. It seems probable that the primary vortices are no longer both in existence during the whole oscillation, but that they are alternately created and annulled, so that when one exists the other does not. If this be so the vortex which exists is that which is on the lee side of the ripple in the state of motion at the instant.

With strong oscillation the secondary vortices apparently do all the work, and the primary vortex, if it exists, only exists for a short time, whilst it may serve as a protecting vortex to the ripple-crest, during the rapid transference of the dividing line of the secondary vortices from one crest to the next. Each secondary vortex is alternately a vortex under the lee of a ripple-mark as exhibited in fig. 10. Mere description is hardly sufficient to explain the motion.

With very violent oscillation the ripple-marks are obliterated, and the water is filled with flying dust.

We now revert to the initiation of ripple-mark.

If the surface be very even, as when sand is sprinkled on glass, when a uniform oscillation of considerable amplitude be established, the sand is carried backwards and forwards and some of the particles stick in places of greater friction. As soon as there is any superficial inequality, it is probable that a vortex is set up in the lee of the inequality which tends to establish a dune there. Such vortices are, however, too small to be seen. The return current in the second half of an oscillation maintains the dune, a vortex being established on the other side, now the lee-side. As the sand tends to stick by friction in a great number of positions, the sand agglomerates into elongated patches, and the patches are so near to one another that the vortex on either side of one patch just fails to interfere with the next patch. As the patches elongate and regularise themselves the vortices increase in intensity, and the vortex established on one lee is not obliterated in the return current. The two vortices are then the primary vortices described above. As the ripple increases in height by the obliteration of some of the elongated patches, the primary vortices set up the secondary vortices. Perhaps the normal state of transition is that only one of the secondary vortices is established at first, and that when the other secondary vortex is set up it tends to obliterate every alternate ripple-mark, and thus to generate a ripple of double wave-length. As the ripples increase in height the secondary vortices become more and more important, and the primary less important. The final or stationary condition is that described above as the case of strong oscillation.

It is to be admitted that this history of the successive stages of the formation of ripple-mark is to some extent speculative, but it is the only method of formation which appears to accord with the various phenomena observed and described above.

It is important to note that when once a fairly regular ripple-mark is established, a wide variability of amplitude in the oscillation is consistent with its maintenance or increase. No explanation of ripple-making can be deemed satisfactory which does not satisfy this condition.

In this summary no attempt has been made to go over again the various peculiarities of the motion, which have been noted above, such as the dance of the crests and of the convolutions of the dividing line of the vortices. We must refer the reader back for the consideration of these points.

§ 3. *The Work of previous Observers and Discussion.*

Some valuable papers on ripple-mark have been lately published.

The first of these is by Mr. A. R. Hunt.* In it he makes an extensive collection of observations on the natural history of ripple-mark. As, however, he does not touch at any length on the mode of formation, I have but little to say on his work. He remarks that regular ripples are due to alternating currents, and that the irregular marks due to currents ought to be distinguished by another name from the regular marks formed by oscillating water. M. Forel, whose paper is referred to below, takes the same view, and describes these irregular marks as dunes. My own observations seem to accord well with the facts collected by Mr. Hunt.

The second paper is by M. Casimir de Candolle.† His experiments have led him to enounce (p. 245) the following general law :—

"When a viscous material in contact with a less viscous liquid experiences an oscillatory or intermittent friction, arising from the relative motion of the liquid layer, 1st, the surface of the viscous material is rippled perpendicularly to the direction of motion; and 2nd, the wave-length is directly proportional to the amplitude of the oscillation."

The word viscous cannot here have its usual meaning, for sand cannot be called viscous. The epithet seems to denote that the constituent parts of the material are mobile, and that there is a considerable amount of internal friction.

When oscillations are set up in a vessel containing two fluids of very unequal viscosity, such as tar and water, ripples are formed on the more viscous fluid. But if the two fluids do not differ widely in viscosity, as mercury and water, water and turpentine, essence of cinnamon and water, ripple-mark is not generated. If, however, a layer of powder be introduced at the surface of separation, ripple-mark is easily formed.

Ripples were made in sand with a variety of fluids, but with olive oil it was found impossible. According to the views maintained in the present paper, the viscosity of oil is too great to permit the generation of the ripple-making vortices.

At p. 257, M. de Candolle writes—

"Chaque ride se termine à la partie supérieure par une crête composée des particules les plus légères. Tant que dure le balancement du liquide, les particules sont animées d'un mouvement pendulaire qui les transporte alternativement de part et d'autre de la crête. Aussi longtemps que l'amplitude de ce balancement est égale à celle qui a donné naissance aux rides, les particules mobiles parcourent à

* "On the Formation of Ripple-mark." Proc. Roy. Soc., April 20, 1882, vol. 34, p. 1.

† "Rides formées," &c. Archives des Sciences Physiques et Naturelles. Genève, No. 3, vol. ix, 15th March, 1883.

chaque demi-oscillation* toute la distance qui sépare l'une de l'autre deux crêtes consécutives. Ce va-et-vient des particules s'étend jusqu'à une certaine distance au-dessous du sommet de chaque crête, mais son amplitude va en diminuant de haut en bas, en raison du poids plus considérable des particules inférieures. Il en résulte que chacune de ces crêtes mobiles a l'apparence d'une lamelle qui oscille sur le sommet de la ride qu'elle termine et s'étire en même temps dans le sens du mouvement de l'eau, ce qui donne tout à fait l'apparence d'un corps visqueux.

"Lorsque l'amplitude du balancement du liquide diminue, il en est naturellement de même des excursions de ces lamelles, et si l'on vient à arrêter subitement le balancement, les particules composant les crêtes mobiles peuvent se déposer entre les rides où elles forment un système de rides secondaires plus minces, intercalées entre celles qui correspondent au maximum d'amplitude du balancement."

In this passage the dance of the particles is in the first place described as being from one side to the other of the ridge, and this, I believe, is the fact. This statement is, however, apparently contradicted by what follows, viz., that the dance is from crest to crest. I have very rarely seen the intercalated ripple-marks to which M. de Candolle refers, but I venture to think that his explanation is not sound, and that they are formed by the particles of sand which, in violent oscillation, have been caught up by the secondary or tree vortices, carried quite round and dropped at the root of the tree, when the oscillations of the water are dying out.

M. de Candolle arrives at the interesting conclusion that the wave-length of ripple-mark is independent of the nature of the oscillating fluid. His suggestion that cirrus clouds are ripple-marks between two aerial currents will be referred to below. The whole paper forms a valuable contribution, and should be read by those who are interested in the subject.

The last paper to which I shall refer is by M. Forel.† He has made extensive observations on ripple-mark, formed both naturally and artificially. He distinguishes between dunes formed by continuous currents either of air or water and ripple-marks formed by oscillation. His view accords with the experiments in § 1 above, but he has not apprehended the importance of the vortex in the lee of the

* In a letter M. de Candolle tells me that "demi-oscillation" should read "quart d'oscillation;" but I still do not see how the ambiguity pointed out below is removed by this correction.

† "Les Rides de Fond." Archives des Sciences Physiques et Naturelles. Genève, 15 Juillet, 1883. M. Forel quotes an article of mine in "Nature" as attributing the formation of ripple-mark to the action of currents in the sea. My statement was intended merely to imply that shallowness of water is favourable to the formation of ripple-mark. I had already made a great part of these experiments when that article was written in 1882.

dune, considering that region merely as slack water. I feel some doubt as to the view that a regular series of dunes may be formed by uniform current; at any rate, in my experiments the dunes were irregular, and had no definite wave-length.

His observations on the circumstances which govern the wave-length of ripple-mark are important. He finds that the factors which enter are the amplitude and period of oscillation of the water; and a third factor is the maximum velocity of the water, which he takes as identical with the ratio between the two others. If the water moves as a whole with a simple harmonic oscillation it is undoubtedly true that the ratio of amplitude to period is proportional to the maximum velocity, but the vortices quite disturb this relation. The maximum velocity of the water relatively to the bottom must depend upon the intensity of the vortices, and this depends upon the height of the ripple-mark.

M. Forel finds that the length and breadth of the vessel have no influence on wave-length, but that it diminishes with increasing depth of water. This he attributes to a diminution both of the period and of the amplitude of the oscillation of the water which is in contact with the bottom. The wave-length increases with the coarseness of the sand.

He remarks that when the ripple-mark is once made, the amplitude of oscillation is without influence on its wave-length. He draws attention to the two limiting velocities, one too great, and the other too small for the formation of ripples, for which values were found in the experiments of § 1.

M. Forel explains ripple-mark as the confluence of two dunes, formed alternately by the oscillating currents. This theory is undoubtedly correct, if somewhat incomplete.

The wave-length, he says, is the amplitude of oscillation of a grain of sand "librement transportée par l'eau." This expression requires further explanation; if it means the amplitude of oscillation of a particle of water at the bottom, when the oscillation is started, and before the ripples have risen, I am disposed to doubt it. It may mean the distance which an average grain of sand is transported when lying on the surface of other sand, under the like circumstances;* if so, ripple-marks formed with a thin layer of sand on a sheet of glass should have a longer wave-length than if the sand be thick; this I am also disposed to doubt. However this may be, M. Forel considers that the wave-length should vary directly as the amplitude of oscillation, directly as the velocity of the current, inversely as the density of the sand, and inversely as the size of the grains. Considering with M. Forel that the velocity of the current is

* I learn by a letter from M. Forel that this is his meaning.

the ratio of amplitude to period, we should have the wave-length for any one sand varying as the frequency multiplied by the square of the amplitude. The few fairly consistent experiments recorded in § 1, do not accord with this view, for it seemed that wave-length varied as v, which is proportional to frequency multiplied by amplitude. This I understand to accord with M. de Candolle's law. As to the law that the finer the sand the longer the wave-length, M. Forel justly observes that it is in contradiction with the fact that small ripples are formed by fine sand, and large ripples by coarse sand. But he endeavours to remove the apparent inconsistency by remarking in effect that the larger limiting velocity for fine sand is smaller than the smaller limiting velocity for coarse sand. There must undoubtedly be truth in this view, but I hesitate to accept it as the whole truth.

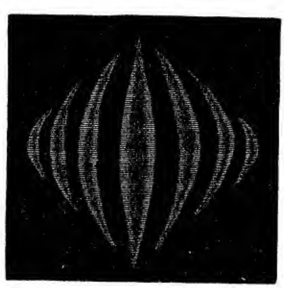
Noticing that, in the same sites in the Lake of Geneva, the ripples are always of the same length, he says, "de ces observations il semblerait résulter que l'intensité des vagues a bien peu d'influence sur la largeur des rides; que la nature du sol est le seul facteur important."

It appears to me that M. Forel's view as to the wave-length of ripple-mark cannot be accepted as final, but he has certainly thrown much light on the subject in his interesting paper.

The following considerations bear upon the laws of wave-length:—

It appeared that in the initial stages of ripple-making, the wave-length is at first only half as long as it becomes ultimately, and that when the layer of sand is thin, the wave-length always remains shorter than if it is thick. Hence if a little sand is dusted on to the oscillating sheet of glass, it is found that the wave-length of ripple is long in the middle of the patch of sand, and short near the margins. Thus the patch when ripple-marked presents such an appearance

Fig. 12.



as fig. 12. If the sand is thin, this appearance often persists however long the oscillation is maintained. This shows that wave-

length is a function of the height of the existing undulations; that is to say, not only of the amplitude of oscillation of the upper part of the vortices, but also of their intensity. On the parts of the plate where the sand is thick, a continual rearrangement of ripple-mark goes on; the wave-length extends by the excision of short patches of intercalated ripple-mark, and by general rearrangement. Finally the sand reaches an ultimate condition as regards wave-length, although rearrangement of ripple-mark still appears to go on for a long time. Then we find in this final condition most of the sand arranged with a certain fundamental wave-length, but where the sand is thin, patches remain with the octave or half wave-length.

It is not easy to understand precisely the mode in which the oscillation of the water over the undulating bottom gives rise to vortices, but there are familiar instances in which nearly the same kind of fluid motion must occur.

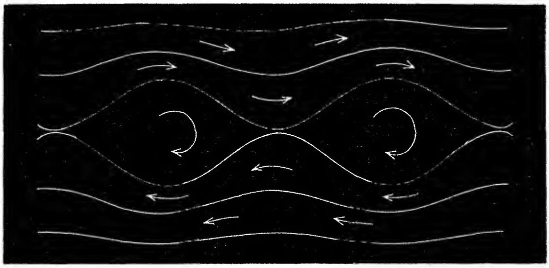
In the mode of boat propulsion called sculling, the sailor places an oar with a flat blade through a rowlock in the stern of the boat, and, keeping the handle high above the rowlock, waves the oar backwards and forwards with an alternate inclination of the blade in one direction and the other. This action generates a stream of water sternwards. The manner in which the blade meets the water is closely similar to that in which the slopes of two ripple-marks alternately meet the oscillating water; the sternward current in one case, and the upward current in the other are due to similar causes. We may feel confident that in sculling, a pair of vortices are formed with axes vertical, and that the dividing line between them is sinuous. The motion of a fish's tail gives rise to a similar rearward current in almost the same way. These instances may help us to realise the formation of the ripple-making vortices.

Lord Rayleigh has considered the problem involved in the oscillations of a layer of vortically moving fluid separating two uniform streams.* At the meeting of the British Association at Swansea in 1880, Sir William Thomson read a paper discussing Lord Rayleigh's problem.† He showed that, in a certain case in which the analytical solution leads to an infinite value, there are waves in the continuous streams in diametrically opposite phases, and that the vortical stratum consists of a series of oval vortices. Fig. 13 illustrates this mode of motion. The uniform current flowing over existing ripple-mark exhibits almost a realisation of this mode of motion, one of the streams of fluid being replaced by the sandy undulations. The same kind of motion must exist in air when a gust of wind blows a shallow puddle into standing ripples.

* "On the Stability or Instability of certain Fluid Motions." Proc. Lond. Math. Soc. (Feb. 12, 1880), vol. xi, p. 57.

† Nature, Nov. 11, 1880, pp. 45–6, and see correction on p. 70.

Fig. 13.



It seems probable that what is called a mackerel sky is an evidence of a closely similar mode of motion; and this agrees with M. de Candolle's suggestion that cirrus is aërial ripple-mark. The layer of transition between two currents of fluid is dynamically unstable, but if a series of vortices be interpolated, so as to form friction rollers as it were, it probably becomes stable. It is likely that in air a mode of motion would be set up by friction, which in frictionless fluid would be stable. If one of the currents of air be colder than the other, a precipitation of vapour will be caused in the vortices, and their shapes will be rendered evident by clouds.*

The direction of striation and velocity of translation of mackerel clouds require consideration according to this theory. If the velocity of the upper current be u, and of the lower current be $-u$, the interposed vortices have a velocity zero, and have their axes perpendicular to the velocity u. Hence if relatively to the earth the rectangular components of the upper current are $u+w$, v, and of the lower are $-u+w$, v, the component velocities of the vortices are w, v, and their axes are parallel to the component v.

Therefore the striations should be parallel to that direction in which the two currents have equal components, and the component velocity of the clouds parallel to the striations should be equal that of either current in the same direction. The resultant velocity of the clouds is clearly equal to a half of that of the two currents, and the component velocity of the striations perpendicular to themselves is the mean of the components of the two currents in the same direction.

[If one of the currents veers the axes of the vortices are shifted.

* I had suggested that the centrifugal force of the vortical rotation must produce rarefaction, fall of temperature, and precipitation of vapour. I have to thank Professor Stokes for pointing out that the fall of temperature would necessarily be so small as only to cause precipitation if the air were almost completely saturated. I think the fall of temperature at the centre of the vortex might be between a hundredth and a fiftieth of a degree Centigrade. [Sir W. Thomson tells me that he had given this vortical explanation before the B. A. in 1876. The volume, however, gives no abstract. He explains the formation of cloud as due to the upward motion in the vortices, and consequent rarefaction.—Jan. 4, 1884.]

The existing clouds will be furrowed obliquely, and the vortical stratum will be cut up into diamond-shaped spaces, determined by the intersection of the old and new vortex-axes. This would explain the patchwork arrangement commonly observed in mackerel sky. May not the lengths of the patches give a measure of the rate of veering of one of the currents ?—Jan. 4, 1884.]

The above account of the formation of ripple-marks shows it to be due to a complex arrangement of vortices. The difficulty of observation is considerable, and perhaps some of the conclusions may require modification. I hope that other experimenters may be induced to examine the question.

Lord Rayleigh has shown me a mathematical paper, as yet unpublished, in which he has considered the formation of aërial vortices over a vibrating plate. It seems possible that an application might be made of similar modes of approximation to the question of water oscillating over a corrugated bottom. Even a very rough solution would probably throw much light on the exact changes which the ripple-making vortices undergo, and any guidance from theory would much facilitate observation.

"On the Atomic Weight of Titanium." By T. E. Thorpe, F.R.S. Received November 7, 1883.

(Preliminary communication.)

The stoïchiometrical quantities which we ordinarily term atomic weights are not only the fundamental constants of chemical calculations; their relations as mere numbers are of the highest significance in connexion with our conceptions concerning the essential nature of matter.

The recent publications of Becker and Clarke in America, and of Lothar Meyer and Seubert in Germany, have served to demonstrate on how slight an experimental basis a large number—the greater proportion, it must be confessed—of the accepted values of these constants really depend.

A notable instance of this fact is seen in the case of titanium. The atomic weight of this element was determined by Rose in 1829, and by Pierre in 1847 with the following results :—

Rose..........	48·13 and 49·58,
Pierre	50·25.

The commonly accepted value of titanium is that founded upon the experiments of Pierre; the atomic weight adopted by Mendelejeff in the series based upon his periodic law is 48, a number which finds

Fig. 1.

FIG. 2.

FIG. 3.

Fig 4.

FIG 5.

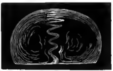


FIG. 6.

FIG. 7.

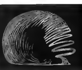


FIG. 8.

FIG. 9.

FIG. 10.

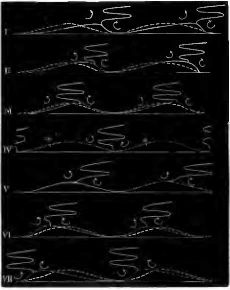


FIG. 11.

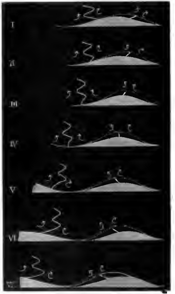


Fig. 12.

Fig. 13.